

CP Asymmetries in Many-Body Final States in Beauty & Charm Transitions

I.I. Bigi^a

^a *Department of Physics, University of Notre Dame du Lac, Notre Dame, IN 46556, USA*
email addresses: ibigi@nd.edu

Abstract

Our community has focused on two-body final states in B & D decays. The SM produces at least the leading source of CP violation in B transitions; none has been established yet in charm decays. It is crucial to measure three- and four-body final states (FS) with accuracy and to compare with predictions based on refined theoretical tools. Correlations between different final states (FS) based on CPT invariance are often not obvious, how to apply them and where. We have to probe *regional* asymmetries and use refined parametrization of the CKM matrix. One uses (broken) U- & V-spin symmetries for spectroscopy. The situations with weak decays of hadrons are much more complex. The impact of strong re-scattering is large, and it connects U- & V-spin symmetries. Drawing diagrams often does not mean we understand the underlying dynamics. I give a few comments about probing the decays of beauty & charm baryons. I discuss the ‘strategies’ more than the ‘tactics’.

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1 Symmetries & tools

We have entered a novel era about probing heavy flavor dynamics, namely to connect "accuracy" and "correlations" on higher levels. The best fitted analyses are often not the best referees. I give a long introduction for several reasons. Often I disagree with statements in published papers about the impact of U-spin symmetry; I will show a list of such articles at appropriate places.

(i) The Standard Model (SM) produces at least the leading source of the measured CP violation (CPV) in neutral kaons and B transitions.

(ii) No CP asymmetry has been established yet in the decays of charm hadrons or baryons in general (except 'our existence').

(iii) The neutral Higgs-like state has been found in the SM predicted mass region without sign of New Dynamics (ND) in its decays; on the other hand that has not been ruled out.

(iv) Neutrino oscillations have been found with $\Delta m(\nu_i) \neq 0$ and three non-zero angles. There is a decent chance to find CP asymmetries there in the future despite the huge asymmetries in nuclei vs. anti-nuclei.

(v) We have completely failed understanding the huge asymmetry in known matter vs. anti-matter in 'our' universe.

Both on the experimental and theoretical sides we have measured CP asymmetries in mostly (pseudo-)two-body final states (FS) in the transitions of K_L and B mesons. However, we have to go beyond them, namely to measure "regional" asymmetries in three- & four-body FS with accuracy. My main points are: those are *not* back-up of the informations we already get from two-body FS: they give us novel information about underlying dynamics, whether about non-perturbative forces of QCD and/or about ND.

We have to discuss the impact of *re-scattering* with some details and how to test our tools. Of course, it needs much more work, but also tells us about fundamental forces. It is a true challenge to deal quantitatively with non-perturbative forces.

Indirect CPV in the neutral mesons transitions ($K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, $B_s^0 - \bar{B}_s^0$ and $D^0 - \bar{D}^0$) can be measured well in two-body FS, and it happened for K^0 and B^0 mesons already with good accuracy.

Direct CP asymmetries in the decays of beauty and charm (& strange) hadrons (& τ leptons) need interferences between amplitudes with differences both in weak and strong phases¹. The first one comes from weak quark dynamics, which is not trivial, but still the easier part of our ‘job’. The second one depends on the impact of QCD. One can use ‘constituent’ quarks that are models for spectroscopy of hadrons. Yet *current* quarks are based on real quantum field theories to describe rates of hadrons. There are subtle, but important differences between the worlds of hadrons and quarks usually named “duality”.

In the SM with three families of quarks CP asymmetries are described by six triangles with different shapes, but all with the same area. Four of those can be probed directly about weak phases:

1. The triangle of $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ has been named the ‘golden’ one in the previous millennium due to large CP asymmetries in B^0 & B^+ transitions.
2. *Indirect* CPV in $B_s \rightarrow \psi\phi, \psi f_0(960)$ is clearly smaller than in $B^0 \rightarrow \psi K_S$ due to $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$, but still not very small: it is still possible around a few percent in B_s^0 transitions.
3. SM produces very small CP asymmetries in singly Cabibbo suppressed amplitudes (SCS) of $D_{(s)}$ due to $V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = 0$ and much less for doubly Cabibbo suppressed (DCS) ones. The latter depends on weak coupling of $V_{cd}^*V_{us}$ in the SM. We know that the 2×2 sub-matrix of $V_{ud}, V_{us}, V_{cd}, V_{cs}$ is not unitary: $\det[V_{2 \times 2}] \neq 1$; however it hardly produces measurable phase as discussed below.
4. Super-tiny rates in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ & $K_L \rightarrow \pi^0 \nu \bar{\nu}$ due to $V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$, where there is very good theoretical control of SM dynamics [1]. There is an experimental challenge to establish these two rates.

One can look at the ‘golden’ triangle with the limits from the measured values of ϵ_K and $\Delta M(B^0)/\Delta M(B_s^0)$ that connects, see Fig.1. Direct CPV has been established in $K_L \rightarrow \pi^+ \pi^-$ vs. $K_L \rightarrow \pi^0 \pi^0$. It is an interesting challenge for LQCD to understand in the future, whether ND could hide there so far [2]. There are others, where we have not enough data to test them like for charm transitions and very rare decays of kaons. The correlations are crucial with other transitions. We are on the first step, but not even close to our goal, and we have to go for precision.

In τ decays we expect simpler landscape in general. The SM predicts basically zero CP asymmetries in τ decays except in FS with the well measured $K^0 - \bar{K}^0$ oscillations

¹I do not include QED corrections; for now they cannot produce real impact for B , D & τ .

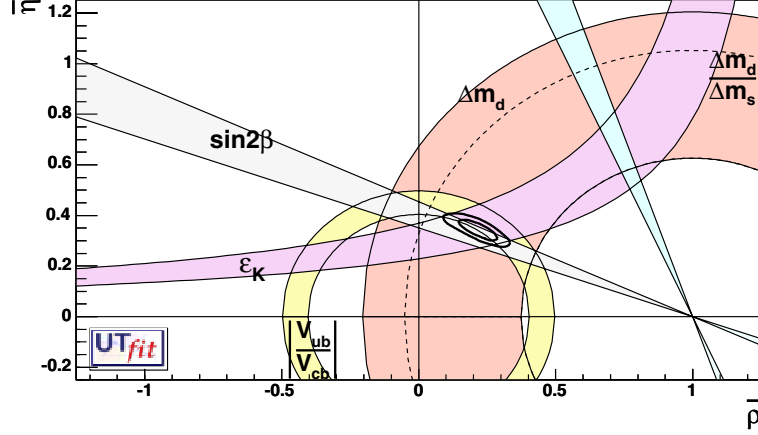


Figure 1: Correlations between other triangles ($\beta = \phi_1$)

(never mind where it comes from). One can use τ decays to calibrate the impact of low energy strong forces with accuracy and well tested theoretical tools like chiral symmetry. Yet surprises can happen.

The situation has changed significantly in the beginning of this millennium. We know that the SM produces at least the leading source of the measured non-zero CP asymmetries. We have to use refined theoretical (& experimental) tools about hadronic forces. In this article I will focus on these items:

(a) We have to probe regional CP asymmetries with accuracy in three- & four-body FS in the decays of charm and beauty hadrons. Beyond differences in averaged vs. regional asymmetries there are subtle questions: how do we *define* "regional" asymmetries and the impact of resonances in general, not only narrow ones?

(b) It is important to measure *correlations* with FS from different decaying beauty & charm (& strange) hadrons. We have to work about these challenges with the best available tools for non-perturbative forces.

(c) It is not surprising to assume CPT invariance. In the worlds of quarks (& gluons) one applies to classes of transitions, not only for total ones. For the worlds of hadrons it is much more subtle in general. Fundamental theories are formulated in the world of quarks.

(d) Lipkin introduced U- & V-spin symmetries as a good tool about *spectroscopy* in hadronic dynamics [3], in particular about light flavor baryons. The situation is much subtle about pions & kaons; chiral symmetry solves the puzzles about their masses.

(e) When one includes weak decays, the dynamics are more complex. For strange hadrons there is hardly a difference between exclusive and inclusive decays. However, there are large one for charm and huge one for beauty hadrons in non-leptonic ones.

It seems to be popular to probe FS based on U-spin symmetry; below I give a list of papers about this item. I disagree with statements like "... model-independent ² relations

²There are subtle differences between being 'insensitive' vs. 'independent' as discussed before; I prefer

are based on U-spin symmetry ...”, in particular about its (semi-)quantitative impact. I will explain that below, why I disagree.

(f) When we discuss weak decays of hadrons, we have to think about the impact of strong *re-scattering* not only in principle, but also (semi-)quantitatively. It is very important to apply other tools in different levels with ”judgment”.

(g) Diagrams with quarks can show us the ‘road’ to understand the underlying dynamics, but not always getting very close to our goal.

(h) Of course, we follow the traditional steps: (i) models; (ii) model-insensitive analyses. However that is not the end of the road: we need another class of steps (iii), namely using refined tools to describe the data with good control. Very often the true theories do *not* give the best fitting of the data; their strengths are based on correlations with other transitions in a ‘network’. I focus on steps (ii) for now and talk about the strategies about steps (iii) for the future.

Before 1990’s Lipkin discussed SCS $D^0 \rightarrow K^+ K^-$ vs. $D^0 \rightarrow K^0 \bar{K}^0$ [4]. Based on tree diagrams with four quarks one gets $\Gamma(D^0 \rightarrow K^0 \bar{K}^0) = 0$. To describe the existing data one needs strong ”final state interaction” (FSI) that obviously comes from non-perturbative QCD forces. Obviously U-spin symmetry is not solid as isospin one by far. In particular, it was pointed out in Ref.[5] that it is not enough to say that a U-spin symmetry takes care of that problem – there are dynamics effects in two-body FS, in particular by FSI; I agree. Lipkin mentioned that $\Gamma(D^0 \rightarrow K^0 K^- \pi^+)$ is *not* balanced by $\Gamma(D^0 \rightarrow K^0 K^+ \pi^-)$ based on U-spin symmetry breaking.

Several of these items were mentioned in 1987 [6] and discussed in 1989 [7], namely the impact of three-body FS. The review ‘The Physics of the B Factories’ [8] gives a broad context including charm & τ decays.

I ‘paint’ the landscape of CP asymmetries about decays of beauty and charm hadrons first with averaged ones and later regional ones. It is easier to apply averaged strong phases to amplitudes for total rates. However the landscape is more complex for CP asymmetries, in particular for regional ones with three- and four-body FS – like the large impact of resonances on probing CP asymmetries.

(i) FSI and ”re-scattering” basically refer to the same strong dynamics mostly by non-perturbative QCD. Often authors use the words of FSI and ”re-scattering” in somewhat different situations, but for my goals about strategies it makes hardly any difference. Therefore I use mostly the word of FSI, but sometimes I use ”re-scattering” to make my point clearer, see Sects. 2.2.1, 2.2.2.

(ii) There are three classes of diagrams:

Class I: Tree diagrams including perturbative QCD corrections are described with the usual symbol ” \rightarrow ”.

Class II: ”Penguin” diagrams connect two quarks Q & q have the same electric charge in their transitions, as I discuss below in Sect. 2.2.2 in three *different* situations of dynamics. Even so, I use the same symbol ” \Rightarrow ” just to remember the reader about these.

Class III: Weak annihilation (WA) [9] diagrams with the symbol ” \Rightarrow ”. When one looks

the word of ‘model-insensitive’.

at with pseudo-scalar $Q\bar{q}$ with tree diagrams, one sees diagrams with weak annihilation or weak exchanges; however QCD corrections mix them, therefore one can use the word of WA in general. (Of course, the number of "colors" has impact.) On the other hand their impact is suppressed by chiral symmetry. The situation for baryons Qq_1q_2 is quite different: they are hardly suppressed by chiral symmetry – but I use the same word "WA".

I see no good reason why two-, three- & four-body FS follow the same pattern; furthermore one should expect differences in beauty & charm transitions. Finally I want to emphasize that we should not just 'trust' diagrams. We have to check them due to "correlations" with other transitions and compare with other symmetries and tools. A well-known example is shown in the Fig. 1 in previous millenary. However the landscape is more complex now. I will discuss: diagrams vs. operators; penguin diagrams vs. FSI and measured rates & asymmetries vs. (anti-)quark amplitudes with FSI. These items are obviously connected.

This paper is organized as follows: Sect. 2 comments about the landscapes of inclusive vs. exclusive decays of charm & beauty hadrons including CPT invariance and (broken) U- & V-spin symmetry; Sects. 3 & 4 focus on beauty & charm hadrons in very different landscapes; a summary is given in Sect. 5.

2 Landscape of about beauty & charm transitions

The connections of the worlds of hadrons and quarks (& gluons) are often not straightforward. Actually there are also differences between the worlds of Hadrodynamics vs. HEP on the theoretical sides. Both of them discuss FSI, but in different landscapes and also where the term "duality" have some different meanings. Obviously I am biased there.

For three-body FS we can use tested tools for probing Dalitz plots and better connect the worlds of hadrons and quarks. Those tools have been used before for strong transitions. Now we have to apply to weak decays of charm and beauty hadrons including low energy collisions of pseudo-scalar states – in particular about the interferences for CP asymmetries. Those three-body FS are greatly affected both by *narrow* & *broad* resonances; the broad ones, in particular about scalar ones, are *not* well described by Breit-Wigner (BW) parametrization. We have to probe four-body FS and go *beyond* their momenta. We can and should follow different 'roads' to this 'Rome'.

I will give more detailed statements. Not all of them are really novel, but often forgotten with their connections. We have to understand the information that the data give us as much as possible. There is also crucial difference between exclusive vs. inclusive rates & asymmetries. Lipkin obviously knew about these differences and showed it [4, 10].

2.1 Refined parametrization of the CKM matrix

Dynamics of flavor violation in the SM world of hadrons are described as the first step by the CKM matrix. It is described by three angles and one weak phase (with three families). Most people use its parametrization going back to Wolfenstein that make its

pattern obvious [11]: with three parameters – A , $\bar{\rho}$ and $\bar{\eta}$ – assumed to be of the order of unitary and known $\lambda \simeq 0.225$ to be used for expansions in higher orders. It describes the data about flavor dynamics quite well including CP violation. There is only one subtle problem: data suggest that $|\bar{\eta}|$ and even more $|\bar{\rho}|$ are not of order unity: $|\bar{\eta}| \simeq 0.34$ and $|\bar{\rho}| \simeq 0.13$. It is surprising how this obvious pattern is so successful despite its disagreement with expected values of $\bar{\eta}$ and $\bar{\rho}$.

Other parameterizations have been suggested for good reasons. One has been found specifically in Ref.[12], when three parameters are truly of the order of unity ($f \sim 0.75$, $\bar{h} \sim 1.35$ and $\delta_{QM} \sim 90^\circ$), while the well-known λ is not. The SM produces at least the leading source of CPV in measured B transitions and predicts very small CPV in D ones:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \quad (1)$$

$$= \begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} - \frac{\lambda^6}{16}, & \lambda, & \bar{h}\lambda^4 e^{-i\delta_{QM}}, \\ -\lambda + \frac{\lambda^5}{2}f^2, & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}(1 + 4f^2) - f\bar{h}\lambda^5 e^{i\delta_{QM}} + \frac{\lambda^6}{16}(4f^2 - 4\bar{h}^2 - 1), & f\lambda^2 + \bar{h}\lambda^3 e^{-i\delta_{QM}} - \frac{\lambda^5}{2}\bar{h}e^{-i\delta_{QM}}, \\ f\lambda^3, & -f\lambda^2 - \bar{h}\lambda^3 e^{i\delta_{QM}} + \frac{\lambda^4}{2}f + \frac{\lambda^6}{8}f, & 1 - \frac{\lambda^4}{2}f^2 - f\bar{h}\lambda^5 e^{-i\delta_{QM}} - \frac{\lambda^6}{2}\bar{h}^2 \end{pmatrix} + \mathcal{O}(\lambda^7) \quad (2)$$

Thus the landscape of the CKM matrix is more subtle now: it is described by six triangles with six different patterns, but still with the same area:

$$\text{Triangle I.1 : } V_{ud}V_{us}^* [\mathcal{O}(\lambda)] + V_{cd}V_{cs}^* [\mathcal{O}(\lambda)] + V_{td}V_{ts}^* [\mathcal{O}(\lambda^{5\&6})] = 0 \quad (3)$$

$$\text{Triangle I.2 : } V_{ud}^*V_{cd} [\mathcal{O}(\lambda)] + V_{us}^*V_{cs} [\mathcal{O}(\lambda)] + V_{ub}^*V_{cb} [\mathcal{O}(\lambda^{6\&7})] = 0 \quad (4)$$

$$\text{Triangle II.1 : } V_{us}V_{ub}^* [\mathcal{O}(\lambda^5)] + V_{cs}V_{cb}^* [\mathcal{O}(\lambda^{2\&3})] + V_{ts}V_{tb}^* [\mathcal{O}(\lambda^2)] = 0 \quad (5)$$

$$\text{Triangle II.2 : } V_{cd}^*V_{td} [\mathcal{O}(\lambda^4)] + V_{cs}^*V_{ts} [\mathcal{O}(\lambda^{2\&3})] + V_{cb}^*V_{tb} [\mathcal{O}(\lambda^{2\&3})] = 0 \quad (6)$$

$$\text{Triangle III.1 : } V_{ud}V_{ub}^* [\mathcal{O}(\lambda^4)] + V_{cd}V_{cb}^* [\mathcal{O}(\lambda^{3\&4})] + V_{td}V_{tb}^* [\mathcal{O}(\lambda^3)] = 0 \quad (7)$$

$$\text{Triangle III.2 : } V_{ud}^*V_{td} [\mathcal{O}(\lambda^3)] + V_{us}^*V_{ts} [\mathcal{O}(\lambda^{3\&4})] + V_{ub}^*V_{tb} [\mathcal{O}(\lambda^4)] = 0 \quad (8)$$

I give two examples with different reasons. (a) In the Wolfenstein parametrization one gets one of large weak phases from V_{td} leading to $\text{Im } V_{td}^*V_{ts} = -A^2\lambda^5\eta$. Now one gets $V_{td}^*V_{ts} = -f\bar{h}\lambda^6\sin\delta_{QM}$ from V_{ts} . (b) The measured indirect CPV in B^0 decays – $S(B^0 \rightarrow \psi K_S) = 0.676 \pm 0.021$ – is close the maximal value that the SM can produce, namely $S(B^0 \rightarrow \psi K_S) \sim 0.72$, which is not close to 100 % [13].

The real challenges come from strong dynamics. QCD is the only local theory that can describe that; however it depends how much we can trust our control over strong forces. To move forward we need more work – and intelligent judgment.

2.2 Theoretical tool kits for many-body FS

Quark diagrams give us two-dimensional plots. However FS with more than three hadrons cannot be described like that in general. The connections of quark diagrams with oper-

ators are subtle, in particular about local vs. non-local operators; the latter depends on long-distances FSI.

(a) Measuring two-body FS gives one-dimensional observables from the rates and only numbers of CP asymmetries. Probing Dalitz plots for CP asymmetries give two-dimensional observables as we have seen already about B decays. One applies amplitudes for FS with hadrons and resonances: $P \rightarrow h_1[h_2h_3] + h_2[h_1h_3] + h_3[h_1h_2] \rightarrow h_1h_2h_3$. I am not claiming that amplitudes of three-body are described by a sum of two-body FS perfectly. To be realistic it is enough for a long time.

(b) It is a good reason to state that the analyses are model-insensitive. We have to measure correlations with other data. Using averaged strong phases in three-body FS (or more) is the first step to understand dynamics. We have the tools to probe Dalitz plots about regional asymmetries. One uses model-insensitive and then uses tools that are checked due to correlations with other transitions as long as they are "acceptable"; the meaning of that depends.

(c) One has to be realistic when probing four-body FS and uses the best tested tools to analyze one-dimensional asymmetries. We are at the beginning to that road to understand the underlying forces.

(d) There are many strengths of lattice QCD, yet re-scattering is not one of those.

There are connections with effective quark operators and hadronic transitions due to ‘duality’ – but they are subtle. As discussed in details in Ref. [14], one cannot compare only the FS using measured masses of hadrons and suggested ones for quarks – it misses the crucial point of duality, namely the impact of *non*-perturbative forces.

The situation of CP asymmetries in beauty & charm hadrons give ‘wonderful challenges’ for probing ND. At least we learn about the impact of FSI in the world of hadrons.

2.2.1 Effective transition amplitudes including FSI

Based on CPT invariance one can describe the amplitudes of hadrons following the history sketched above; it is given in Refs.[15, 16, 17] and in Sect. 4.10 of Ref.[18] with much more details:

$$T(P \rightarrow f) = e^{i\delta_f} \left[T_f + \sum_{f \neq a_j} T_{a_j} i T_{a_j f}^{\text{resc}} \right] \quad (9)$$

$$T(\bar{P} \rightarrow \bar{f}) = e^{i\delta_f} \left[T_f^* + \sum_{f \neq a_j} T_{a_j}^* i T_{a_j f}^{\text{resc}} \right] ; \quad (10)$$

$T_{a_j f}^{\text{resc}}$ describe FSI between f and intermediate on-shell states a_j that connect with this FS. It points out that f are different from a_j , but are in the same classes of strong dynamics. In the world of quarks one describes $a_j = \bar{q}_j q_j$ and $f = \bar{q}_k q_k + \text{pairs of } \bar{q}_l q_l \text{ with } q_{j,k,l} = u, d, s$.³

³In the worlds of Hadrodynamics one can use basically the same equations, where the actors are $\pi\pi \rightarrow \bar{K}K$ plus pairs of π etc. One sees there are different cultures in Hadrodynamics vs. HEP.

One gets regional CP asymmetries, not just averaged ones:

$$\Delta\gamma(f) = |T(\bar{P} \rightarrow \bar{f})|^2 - |T(P \rightarrow f)|^2 = 4 \sum_{f \neq a_j} T_{a_j f}^{\text{resc}} \text{Im} T_f^* T_{a_j} ; \quad (11)$$

these FS f consist of two-, three-, four-body etc. pseudo-scalars. In principle one can probe local asymmetries – but one has to be realistic with finite data and a lack of ‘perfect’ quantitative control of non-perturbative QCD. We need real theories about understanding of SM & ND dynamics. We test our understanding of the information due to correlations with the data in ”acceptable” ways. This statement is subtle, but crucial; I will discuss them in some details.

CP asymmetries have to vanish upon summing over all such states f using CPT invariance between subclasses of partial widths:

$$\sum_f \Delta\gamma(f) = 4 \sum_f \sum_{f \neq a_j} T_{a_j f}^{\text{resc}} \text{Im} T_f^* T_{a_j} = 0 , \quad (12)$$

since $T_{a_j f}^{\text{resc}}$ & $\text{Im} T_f^* T_{a_j}$ are symmetric & antisymmetric, respectively, in the indices f & a_j .

It is one thing to draw quark diagrams by adding pair of $\bar{q}q$, but it is quite another thing to trust them. How can one connect the data about the decays with two-, three-, four-body FS with the information about the underlying dynamics? We have to apply several theoretical tools there, connect with others transitions and think about their limits. I will discuss U-spin symmetry & its uncertainties in some details for good reasons and its connections with V-spin one. To make the important statement with somewhat different words: I see no reason why $T_{a_j f}^{\text{resc}}$ do not connect U- & V-spin transitions as pointed out just above. I will discuss below in some details for beauty & charm decays.

There is a crucial point to understand that, namely the connection of the world of measured rates & their asymmetries and the amplitudes one gets from the dynamics of quantum field theories as you can see from Eqs. (9,10). It is not one-to-one as one sees by diagrams – it is more subtle. FSI happen all the times and often with sizable impact. The question is: how much and where. We need help from other tools like chiral & G-parity symmetries, dispersion relations etc. to understand the information given by the data; and the item of ”duality” [19] comes in different situations.

2.2.2 ‘Painting’ the landscape of diagrams including penguin ones

Diagrams can be classified in three classes: (a) Some diagrams describe lifetimes of beauty hadrons etc. with *local* operators based on OPE and HQE [19]. (b) Others discuss *short* distances forces like $\Delta\Gamma(B_s^0)$ (or hadronic jets) [20]. (c) Finally others try to deal with *long* distance dynamics like re-scattering about $\bar{q}_i q_i \leftrightarrow \bar{q}_j q_j$; actually it is for important for $\bar{q}_i q_i \rightarrow \bar{q}_j q_j + \bar{q}_k q_k \bar{q}_l q_l + \dots$

Specifically I comment about penguin diagrams. Those are used to describe their impact on the decays of beauty & charm hadrons, where FS consist mostly of many-body hadrons, not from two-body FS or even not (pseudo-)two-body ones. Therefore I use the word of ‘painting’ the landscape of penguins: the situations are much more complex and

also different for beauty and charm hadrons; furthermore I see no reason why two-, three- and four-body FS follow the same pattern.

Penguin diagrams were introduced by M. Shifman, A. Vainshtein and V. Zakharov in 1975 [21] to explain the measured $\Delta I(3/2) \ll \Delta I(1/2)$ amplitudes in kaon decays and later predicted the direct CP violation $\epsilon'/\epsilon_K \neq 0$. These are based on *local* operators with two-body FS, although they come from loop diagrams.

Inclusive decays of *beauty* hadrons show impact of penguin diagrams in two classes of flavor changes – $b \Rightarrow s$ and $b \Rightarrow d$ – in a calculated way, but hardly for exclusive ones. One can add pairs of $\bar{q}q$ to penguin (or tree) diagrams and claims to produce the numbers of hadrons one wants. However to draw a diagram is one thing, but to describe the process is quite another thing ⁴. The connections of penguin and tree diagrams with reality are often fuzzy as pointed out in Refs. [15, 16, 20]. The name of ‘penguin’ diagrams is often used in a very broad sense: $Q\bar{q}_a \rightarrow q(\bar{q}_i q_i + \bar{q}_i q_i \bar{q}_j q_j + \dots)\bar{q}_a$ following unitarity, where Q and q quarks carry the same charge with or without local operators. Of course, one should not hide theoretical uncertainties. Penguin diagrams give also imaginary part that one needs for FSI [15, 18]. Non-local penguin operator with internal charm lines can produce FSI: with $2m_c < m_b$ they can be *on-shell* and thus produce an imaginary part. However they give us pictorials, but not much more. In the world of hadrons one measures rates and asymmetries, while in the world of (anti-)quarks one produces amplitudes that have to include re-scattering, as pointed out above in Eq.(11). That connection is far from trivial, and we are not yet at the end of the road to understand the real dynamics.

2.2.3 Connect U- & V-spin symmetries: spectroscopy vs. weak decays

The global $SU(3)_{fl}$ was introduced first with its three $SU(2)_I$, $SU(2)_U$ and $SU(2)_V$, when quarks were seen mostly as a mathematical tool to describe the spectroscopies of hadrons, not as real physical states [3]. Therefore ‘constitute’ quarks were used. It is easier to discuss the masses of baryons than those of the mesons, since the latter are greatly affected by chiral symmetry. When one compares the masses of nucleons, Λ and Ξ , one suggests the values of u & d constituent quarks ~ 330 MeV and for s one ~ 500 MeV. We have a better understanding of that due to mixing of $\langle 0|\bar{u}u|0\rangle$, $\langle 0|\bar{d}d|0\rangle$ between $\langle 0|\bar{s}s|0\rangle$ with scalar resonances that are not OZI suppressed [22].

It makes sense to use U-spin symmetry about *spectroscopy* of beauty & charm hadrons. However the situation is much more complex in general: FSI have important impact on weak amplitudes in general and in particular for CP asymmetries. Therefore we cannot ignore the correlations with V-spin symmetry. To say it differently: we cannot focus only on two-body FS and even more with only charged ones in weak transitions. As shown in Eqs.(9,10, 11) intermediate two states strongly re-scatter into FS with two-, three-, four-, ... states either in the world of hadrons or quarks. The main point is very general: there are very different time scales of weak vs. strong forces. Therefore re-scattering makes the

⁴It was predicted 1500 years ago by Marinus, student of Proklos (known Neoplatonist philosopher influential on Western medieval philosophy as well as Islamic thought): "Only *being* good is one thing – but good *doing* it is the other one!"

differences between U- & V-spin symmetries very ‘fuzzy’; actually they are connected as pointed out above in Sect.2.2.1.

The problem is to deal with FSI quantitatively. Obviously U-spin symmetry is sizably broken. One guess is $(M_K^2 - M_\pi^2) < (M_K^2 + M_\pi^2)$. More refined ones are based on the item of ‘constitute’ quarks and give $m_u^{\text{const}} \sim m_d^{\text{const}} < m_s^{\text{const}}$. One can use that for models to predict *exclusive* decays, but with large theoretical uncertainties.

In the world of quarks one describes mostly inclusive transitions. ”Currents” quarks with $m_u < m_d \ll m_s$ are based on theory. I-, U- & V-spin symmetries deal with $u \leftrightarrow d$, $d \leftrightarrow s$ & $u \leftrightarrow s$. These three symmetries are obviously broken on different levels, and these violations are connected in the SM. The operators producing inclusive FS depend on their CKM parameters and the current quark masses involved there. However the real scale for inclusive decays is given by the impact of QCD, namely $\bar{\Lambda} \sim 0.7 - 1$ GeV as discussed many times ⁵. Thus the violations of U- & V-spin symmetries are small, and tiny for I-spin one. We can deal with inclusive rates and asymmetries of beauty and maybe charm hadrons using effective operators in the world of quarks.

The connections with inclusive with exclusive hadronic rates are not obvious at least, in particular about quantitative ways. The violations of I-, U- & V-spin symmetries in the measurable world of hadrons are expected to scale by the differences in pion and kaon masses, which are *not* small compared to $\bar{\Lambda}$ (or $[m_K^2 - m_\pi^2]/[m_K^2 + m_\pi^2]$). This is even more crucial about direct CP violation and the impact of FSI on amplitudes. One reason is that suppressed decays in the world of hadrons consist with larger numbers of states in the FS, where FSI have great impact with opposite signs. Furthermore the worlds of hadrons are controlled by FSI due to *non*-perturbative QCD; they show the strongest impact on exclusive ones. It can be seen in the sum of exclusive ones in large ratios that go up and down much more sizably; I will show well-known examples of that below. However the rules are not yet well established how to connect the worlds of Hadrodynamics and HEP.

2.3 Three-body FS in the decays of beauty & charm hadrons

We have to probe three-body FS with accuracy. The first step is to measure averaged CP asymmetries there and analyze about connections with two-body FS. Then one probes regional CP asymmetries using different technologies for Dalitz plots. Ratios of regional asymmetries do not depend on production asymmetries. One gets more observables to check experimental uncertainties. On the theory side one needs much more work, but check theoretical uncertainties about the impact of non-perturbative QCD and the impact of the existence of ND and its features. We have seen that FSI have large impact, in particular for suppressed decays of beauty and charm hadrons.

There is a good reason to analyze data with model-insensitive ways as the second step. It does not mean there is only one road, actually there are three classes discussed. There is no ‘golden’ tool, when one search for the impact of ND: one uses refined CKM matrix and probes Dalitz plots with different ‘roads’ and compare their results. One puts them

⁵For good reasons one uses different and smaller $\Lambda_{\text{QCD}} \sim 0.1 - 0.3$ GeV for describing jets in collisions.

into small bin i , and customarily one measures ‘fractional asymmetries’ $\Delta(i) \equiv \frac{N(i) - \bar{N}(i)}{N(i) + \bar{N}(i)}$ or ‘Miranda’ procedure [23, 24, 25] uses ‘significance’ $S_{CP}(i) \equiv \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}}$ ⁶; or one can also probe *unbinned* Dalitz plots [26]. These tools mentioned above do that in different ways and compare their results. At least it helps our thinking.

It is not the final step. I want to emphasize the impact of systematical ones in several directions. Furthermore one has to think about correlations with other amplitudes; it needs much more work, but also gives us an ‘award’. BW parameterizations do not well describe the impact of scalar resonances like $f_0(500)/\sigma$ & $K^*(800)/\kappa$ both in charm and beauty hadronic FS. Furthermore we have to discuss about the lists of resonances that are included with finite data. Then we cannot focus only on regional CP asymmetries ‘inside’ narrow resonances with low masses, but also probe ‘outside’ asymmetries. Obviously we can use chiral symmetry to probe the FS. There is a subtle tool, namely refined dispersion relations [27, 22]; they are based on data with low energy collisions of pions and kaons. In the end we learn a lot about underlying dynamics. It is true there are prices: (a) One needs much more data & analyses. (b) We cannot use the same tool kit for several transitions, as you can see in the references [27, 22]. The landscape is complex; however we should not give up. The basic elements for three-body FS: $\pi^+\pi^- \longleftrightarrow \pi^0\pi^0$, $K^+K^- \longleftrightarrow \bar{K}^0K^0$, $K^+K^- \longleftrightarrow \pi^+\pi^-$ & $K^-\pi^+ \longleftrightarrow \bar{K}^0\pi^0$.

For the two-dimensional Dalitz plots we have the tools to probe them with a long history in strong dynamics. One needs larger amounts of data and experimental work, but they also deliver information about the existence of ND and its features. In the end we have to agree after discussions; at least they tell us new lessons about strong forces.

2.4 Four-body FS with different ‘roads’ to ND

It was first pointed out that four-body FS can and should be probed in special situations, namely $B \rightarrow V_1 V_2 \rightarrow h_1 h_2 h_3 h_4$ with V_i describing vector resonances [28]. It was realized that we have a general situations in the decays of heavy flavor hadrons with four-body FS [29]. I will discuss these four-body FS about $\Delta B \neq 0 \neq \Delta C$ below.

Traditionally one compares *T-odd moments* of $H_Q \rightarrow h_1 h_2 h_3 h_4$ vs. $\bar{H}_Q \rightarrow \bar{h}_1 \bar{h}_2 \bar{h}_3 \bar{h}_4$ in centre-of-mass frame: $C_T \equiv \vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$, $\bar{C}_T \equiv \vec{\bar{p}}_1 \cdot (\vec{\bar{p}}_2 \times \vec{\bar{p}}_3)$ leading to *T-odd* observables:

$$A_T \equiv \frac{\Gamma_{H_Q}(C_T > 0) - \Gamma_{H_Q}(C_T < 0)}{\Gamma_{H_Q}(C_T > 0) + \Gamma_{H_Q}(C_T < 0)}, \quad \bar{A}_T \equiv \frac{\Gamma_{\bar{H}_Q}(-\bar{C}_T > 0) - \Gamma_{\bar{H}_Q}(-\bar{C}_T < 0)}{\Gamma_{\bar{H}_Q}(-\bar{C}_T > 0) + \Gamma_{\bar{H}_Q}(-\bar{C}_T < 0)} \quad (13)$$

FSI can produce $A_T, \bar{A}_T \neq 0$ without CPV. However

$$a_{CPV}^{T-odd} \equiv \frac{1}{2}(A_T - \bar{A}_T) \quad (14)$$

would establish CP asymmetry. With more data & more thinking we might have some

⁶Obviously I might be biased about the strengths of these tools.

ideas about ‘better’ values of $d > 0$ that do not depend only on experimental reasons:

$$\begin{aligned} A_T(d) &\equiv \frac{\Gamma_{H_Q}(C_T > d) - \Gamma_{H_Q}(C_T < -d)}{\Gamma_{H_Q}(C_T > d) + \Gamma_{H_Q}(C_T < -d)} \\ \bar{A}_T(d) &\equiv \frac{\Gamma_{\bar{H}_Q}(-\bar{C}_T > d) - \Gamma_{\bar{H}_Q}(-\bar{C}_T < -d)}{\Gamma_{\bar{H}_Q}(-\bar{C}_T > d) + \Gamma_{\bar{H}_Q}(-\bar{C}_T < -d)} \end{aligned} \quad (15)$$

However we cannot stop there – we need one-dimensional observables (at least); furthermore we have to understand the reasons why different observables are used and compare their impact. I give two examples.

- One can measure the angle ϕ between the planes of $h_1 h_2$ and $h_3 h_4$ and described its dependence [18, 30]:

$$\frac{d\Gamma}{d\phi}(H_Q \rightarrow h_1 h_2 h_3 h_4) = \Gamma_1 \cos^2 \phi + \Gamma_2 \sin^2 \phi + \Gamma_3 \cos \phi \sin \phi \quad (16)$$

$$\frac{d\Gamma}{d\phi}(\bar{H}_Q \rightarrow \bar{h}_1 \bar{h}_2 \bar{h}_3 \bar{h}_4) = \bar{\Gamma}_1 \cos^2 \phi + \bar{\Gamma}_2 \sin^2 \phi - \bar{\Gamma}_3 \cos \phi \sin \phi \quad (17)$$

The partial width for $H_Q[\bar{H}_Q] \rightarrow h_1 h_2 h_3 h_4[\bar{h}_1 \bar{h}_2 \bar{h}_3 \bar{h}_4]$ is given by $\Gamma_{1,2}[\bar{\Gamma}_{1,2}]$: $\Gamma_1 \neq \bar{\Gamma}_1$ and/or $\Gamma_2 \neq \bar{\Gamma}_2$ represents direct CPV in the partial width:

$$\Gamma(H_Q \rightarrow h_1 h_2 h_3 h_4) = \frac{\pi}{2}(\Gamma_1 + \Gamma_2) \quad \text{vs.} \quad \Gamma(\bar{H}_Q \rightarrow \bar{h}_1 \bar{h}_2 \bar{h}_3 \bar{h}_4) = \frac{\pi}{2}(\bar{\Gamma}_1 + \bar{\Gamma}_2) \quad (18)$$

Γ_3 and $\bar{\Gamma}_3$ represent T *odd* correlations; however [29, 18]: $\Gamma_3 \neq \bar{\Gamma}_3 \rightarrow$ CPV. Integrated rates give $\Gamma_1 + \Gamma_2$ vs. $\bar{\Gamma}_1 + \bar{\Gamma}_2$; the moments of integrated *forward-backward* asymmetry

$$\langle A \rangle = \frac{\Gamma_3 - \bar{\Gamma}_3}{\pi(\Gamma_1 + \Gamma_2 + \bar{\Gamma}_1 + \bar{\Gamma}_2)} \quad (19)$$

gives information about CPV. When one has enough data to do that, one could disentangle Γ_1 vs. $\bar{\Gamma}_1$ and Γ_2 vs. $\bar{\Gamma}_2$ by tracking the distribution in ϕ . If there is a *production* asymmetry, it gives global $\Gamma_1 = c\bar{\Gamma}_1$, $\Gamma_s = c\bar{\Gamma}_2$ and $\Gamma_3 = -c\bar{\Gamma}_3$ with *global* $c \neq 1$.

- We have learnt from the history of $K_L \rightarrow \pi^+ \pi^- \gamma^* \rightarrow \pi^+ \pi^- e^+ e^-$, where Seghal [31, 32] really predicted CPV there around 14 % based on $\epsilon_K \simeq 0.002$. Of course, the landscapes of $\Delta S \neq 0$ and $\Delta B \neq 0 \neq \Delta C$ are quite different for several reasons; I mention only one now: $\Delta S \neq 0$ amplitudes are described with local operator, but not for the others.

It helps to discuss that situation in more details with unit vectors:

$$\vec{n}_\pi = \frac{\vec{p}_+ \times \vec{p}_-}{|\vec{p}_+ \times \vec{p}_-|}, \quad \vec{n}_l = \frac{\vec{k}_+ \times \vec{k}_-}{|\vec{k}_+ \times \vec{k}_-|}, \quad \vec{z} = \frac{\vec{p}_+ + \vec{p}_-}{|\vec{p}_+ + \vec{p}_-|} \quad (20)$$

$$\sin\phi = (\vec{n}_\pi \times \vec{n}_l) \cdot \vec{z} [CP = -, T = -] \quad , \quad \cos\phi = \vec{n}_\pi \cdot \vec{n}_l [CP = +, T = +] \quad (21)$$

$$\frac{d\Gamma}{d\phi} \sim 1 - (Z_3 \cos 2\phi + Z_1 \sin 2\phi) \quad (22)$$

Then one measures asymmetry in the moments:

$$\mathcal{A}_\phi = \frac{(\int_0^{\pi/2} - \int_{\pi/2}^\pi + \int_\pi^{3\pi/2} - \int_{3\pi/2}^{2\pi}) \frac{d\Gamma}{d\phi}}{(\int_0^{\pi/2} + \int_{\pi/2}^\pi + \int_\pi^{3\pi/2} + \int_{3\pi/2}^{2\pi}) \frac{d\Gamma}{d\phi}} \quad (23)$$

There is an obvious reason to probe only the angle between the two $\pi^+\pi^-$ & e^+e^- planes. It is based on $K_L \rightarrow \pi^+\pi^-\gamma^*$ or $K^0 \rightarrow \pi^+\pi^-\gamma^*$ vs. $\bar{K}^0 \rightarrow \pi^+\pi^-\gamma^*$.

Here the situations are more complex for several reasons; therefore I use:

$$\begin{aligned} \frac{d}{d\phi} \Gamma(H_Q \rightarrow h_1 h_2 h_3 h_4) &= |c_Q|^2 - [b_Q \cos 2\phi + a_Q \sin 2\phi] = \\ &= |c_Q|^2 - [b_Q (2\cos^2\phi - 1) + 2a_Q \sin\phi \cos\phi] \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{d}{d\phi} \Gamma(\bar{H}_Q \rightarrow \bar{h}_1 \bar{h}_2 \bar{h}_3 \bar{h}_4) &= |\bar{c}_Q|^2 - [\bar{b}_Q \cos 2\phi - \bar{a}_Q \sin 2\phi] = \\ &= |\bar{c}_Q|^2 - [\bar{b}_Q (2\cos^2\phi - 1) - 2\bar{a}_Q \sin\phi \cos\phi] \end{aligned} \quad (25)$$

Obviously the landscapes are complex

$$\Gamma(H_Q \rightarrow h_1 h_2 h_3 h_4) = |c_Q|^2 \quad \text{vs.} \quad \Gamma(\bar{H}_Q \rightarrow \bar{h}_1 \bar{h}_2 \bar{h}_3 \bar{h}_4) = |\bar{c}_Q|^2 \quad (26)$$

For these moments one gets:

$$\langle A_{\text{CPV}}^Q \rangle = \frac{2(a_Q - \bar{a}_Q)}{|c_Q|^2 + |\bar{c}_Q|^2} ; \quad (27)$$

i.e., no impact from b_Q & \bar{b}_Q terms.

Furthermore one wants to probe semi-regional asymmetries like:

$$A_{\text{CPV}}^Q|_e^f = \frac{\int_e^f d\phi \frac{d\Gamma}{d\phi} - \int_e^f d\phi \frac{d\bar{\Gamma}}{d\phi}}{\int_e^f d\phi \frac{d\Gamma}{d\phi} + \int_e^f d\phi \frac{d\bar{\Gamma}}{d\phi}} \quad (28)$$

where b_Q and \bar{b}_Q contribute. Again, the main point is not to choose which gives the best fitting one, but has deeper reasons. I will discuss more specifically later.

These examples are correct from the general theoretical bases. However, some deal better with experimental uncertainties, cuts and/or probe the impact of ND; also the true underlying dynamics do not always – actually often not – produce the best fitting of the data. It is crucial to use CPT invariance as a tool for correlations with other transitions. It is important to probe semi-regional asymmetries and compare results with different tools, but also with more thinking. Somewhat similar statements can be seen in [33].

2.5 Short resume

My points are about CP asymmetries in the weak decays of charm and beauty hadrons:

- First one probes two-body FS for non-leptonic transitions. However one has to go beyond that, since many-body FS give us much more information about the underlying dynamics. I see no reason, why many-bodies FS follow the same pattern. Actually present data show different patterns, as I show below.
- I suggest there are the same classes I, II & III for amplitudes for two-, three- & four-body FS. However the goals are quite different, namely leading sources for $\Delta C \neq 0$ or close to them, but non-leading ones for $\Delta B \neq 0$.
- Indirect CPV is mostly best measured in time dependent rates with two-body FS.
- CPT invariance produces much more equalities in the masses & widths of particles and anti-particles: equalities of sub-classes of decays are defined by FSI due to strong dynamics, see Eqs. (9, 10, 11). There are correlations between two-, three-, four-body FS that often are not obvious.
- A ‘popular’ tool is used to connect different FS, namely (broken) U-spin symmetry. However we have to deal with more complex situations:
 - One can apply U-spin symmetry to discuss hadronic *spectroscopy* with decent uncertainties. However the landscapes have changed significantly, when one includes *weak* dynamics. Re-scattering connect U- & V-spin symmetries. Below I will discuss statements given in a list of papers based on U-spin symmetry and explain why I disagree. Those papers had just ignored previous published papers without explaining why.
 - Even in the world of quark diagrams one cannot ignore re-scattering $\bar{q}_i q_i \rightarrow \bar{q}_j q_j + \bar{q}_j q_j \bar{q}_k q_k \dots$. Furthermore we cannot describe that with local operators in general – we need long distance forces.
 - There are crucial differences between ”inclusive” vs. ”exclusive” decays: we learn from the data about the impact of underlying both weak and strong dynamics. One can use expansions of $(m_s - m_d)/m_c$ for ”inclusive” one about U-spin symmetry and likewise for $(m_s - m_u)/m_c$ about V-spin one. Both of them are small. However, the landscape is changing sizably, when we discuss about ”exclusive” ones.
 - We cannot focus only on (quasi-)two-body FS like 2π , $\pi\rho$, 2ρ , $\bar{K}K$, $\bar{K}K^*$ etc.
- Production asymmetries in pp collisions obviously affect in particular charm and beauty *baryons* decays. On the other hand, class III amplitudes can contribute sizably here, since WA [9] diagrams of beauty & charm baryons are not suppressed mostly by chiral symmetry in opposite what happens for meson decays. It would be a great achievement by the LHCb collab. to establish CPV in heavy flavor baryons’ decays no matter where it comes from. I will comments mostly about $\Lambda_c^+ \rightarrow pK^+\pi^-$.

- One has to include scalar resonances that usually are broad ones and are not well described by BW parameterization. With finite data one has to think and discuss about the lists of the resonances included.
- The best fitted analyses do often not give the best understanding of the dynamics.
- I see no reason, why the impact of FSI should follow the same pattern.

We have to discuss correlations with other transitions. Good judgment understands where theories (even theorists) help.

To describe the decays of beauty & charm hadrons it remembers me of the Austrian saying: "It is the same – only different!" The same classes of tools can be used – differently.

3 $\Delta B \neq 0$ forces

The measured lifetimes of B^0 and B_s^0 are the same within 2% as a sign of U-spin symmetry about *inclusive* transitions; likewise for the inclusive semi-leptonic decays. Furthermore they agree the theoretical predictions within those uncertainties.

However the landscapes for *exclusive* non-leptonic decays are different, as you see in PDG2015:

$$\text{BR}(B^0 \rightarrow K^+\pi^-) = (1.96 \pm 0.05) \cdot 10^{-5} \quad (29)$$

$$\text{BR}(B_s^0 \rightarrow K^-\pi^+) = (0.55 \pm 0.06) \cdot 10^{-5} \quad (30)$$

In principle those branching ratios are ‘expected’, since penguin diagrams $b \Rightarrow d$ are more suppressed for B_s^0 transitions. On the other hand $b \Rightarrow d$ produces large weak phases. Therefore we are not surprised by the data:

$$A_{CP}(B^0 \rightarrow K^+\pi^-) = -0.082 \pm 0.006, \quad A_{CP}(B_s^0 \rightarrow K^+\pi^-) = +0.263 \pm 0.035 \quad (31)$$

Based on U-spin symmetry it was suggested in Refs.[34, 10, 35] to probe

$$\Delta = \frac{A_{CP}(B^0 \rightarrow K^+\pi^-)}{A_{CP}(B_s^0 \rightarrow K^+\pi^-)} + \frac{\text{BR}(B_s^0 \rightarrow K^-\pi^+) \tau_d}{\text{BR}(B^0 \rightarrow K^+\pi^-) \tau_s} = 0 \quad (32)$$

Actually it depends only on two-body FS with charged kaon & pion:

$$\Delta = \frac{A_{CP}(B^0 \rightarrow K^+\pi^-)}{A_{CP}(B_s^0 \rightarrow K^+\pi^-)} + \frac{\Gamma(B_s^0 \rightarrow K^-\pi^+)}{\Gamma(B^0 \rightarrow K^+\pi^-)} \quad (33)$$

Present data from LHCb give[36]:

$$\Delta = -0.02 \pm 0.05 \pm 0.04. \quad (34)$$

It is not clear what the data tell us: is it $\Delta \simeq 0$ – the strength of U-spin symmetry – or $\Delta \sim -0.1$ – the impact of re-scattering? To say it with different words about important

questions. On which scale is U-spin symmetry broken? Can one focus only on charged two-body FS? We cannot ignore re-scattering, namely $K^-\pi^+ \Leftrightarrow \bar{K}^0\pi^0$ etc. etc.; more importantly, as I said just above: $(s\bar{u})(u\bar{d}) \rightarrow \bar{K}\pi's$, $\bar{K}K\bar{K}\pi's$ and even $(s\bar{q})(q\bar{d}) \rightarrow \bar{K}\pi's$, $\bar{K}K\bar{K}\pi's$ with $q = u, d, s$. One expects larger impact of penguin diagrams for B^0 vs. B_s^0 transitions, at least in the SM: $\Gamma(B^0 \rightarrow \bar{K}\pi's/\bar{K}K\bar{K}\pi's) > \Gamma(B_s^0 \rightarrow \bar{K}\pi's/\bar{K}K\bar{K}\pi's)$. We know that U- & V-spin symmetries are connected with strong forces, as discussed in general in Sects. 2.2.1, 2.2.2 & 2.2.3.

It has been stated already in the Abstract of the Ref.[35]: "pure-penguin decay $B_s^0 \rightarrow \bar{K}^0 K^0 \bar{K}^0$ is intriguing ..."; I have to disagree. The connections of quark diagrams with hadronic amplitudes is "complex". Below I will discuss in general about B^\pm transitions. Here I talk about this special example: one looks at tree operators $b \rightarrow u(d\bar{u})$ & $b \rightarrow d(u\bar{u})$ and penguin one $b \Rightarrow d$ and embanked into the $B_s^0 = [\bar{b}s]$ wave function. The authors of Ref.[35] said: when one describes three-body FS in SM suppressed B_s^0 decays, one needs only one pair of $\bar{q}q$ for tree diagrams and two pairs of $\bar{q}_1 q_1 \bar{q}_2 q_2$. Therefore tree diagrams produce only $B_s^0 \rightarrow \bar{K}^0 K^+ K^-$, but not $B_s^0 \rightarrow \bar{K}^0 K^0 \bar{K}^0$. I disagree with such statements, since re-scattering with strong forces produces $\bar{u}u \rightarrow \bar{d}d$ or $\bar{u}u \rightarrow \bar{s}s$ and also tree diagrams give $B_s^0 \rightarrow \bar{K}^0 K^0 \bar{K}^0$ in principle, although we cannot give a semi-quantitative production. On the other hand, we have two examples for CP asymmetries in three-body FS of suppressed B^\pm decays, where we found very interesting data, as I discuss below.

3.1 CP asymmetries in B^\pm with CPT invariance

LHCb data show that CKM suppressed B decays mostly populate the boundaries of Dalitz plots. At the qualitative level one should not be surprised. The 'centers' are not really empty. CP asymmetries come from interferences; therefore one expects large regional ones – but how much and where? The impact of broad resonances might be best seen in asymmetries rather than rates.

When one discusses CKM suppressed B^- amplitudes in the SM, one starts with local operators based on left-handed currents $[\bar{u}\gamma_\mu(1-\gamma_5)b][\bar{q}\gamma_\mu(1-\gamma_5)u]$ (with $q = s, d$) leading to renormalized operators

$$O_\pm \propto \{[\bar{u}\gamma_\mu(1-\gamma_5)b][\bar{q}\gamma_\mu(1-\gamma_5)u] \pm [\bar{q}\gamma_\mu(1-\gamma_5)b][\bar{u}\gamma_\mu(1-\gamma_5)u]\}. \quad (35)$$

A local penguin operator O_{Peng} $b \Rightarrow q$ ($q = s, d$) connects a left-handed current with a vector one due to QCD. Thus one gets for the B^- amplitude for $c_+ \langle u\bar{u}s\bar{u}|O_+|B^- \rangle + c_- \langle u\bar{u}s\bar{u}|O_-|B^- \rangle + c_P \langle u\bar{u}s\bar{u}|O_{Peng}|B^- \rangle$. CPT invariance gives:

$$\Gamma(B^- \rightarrow \bar{K} + X_{S=0}) = \Gamma(B^+ \rightarrow K + \bar{X}_{S=0}) \quad (36)$$

$$\Gamma(B^- \rightarrow X'_{S=0}) = \Gamma(B^+ \rightarrow \bar{X}'_{S=0}) \quad (37)$$

The X , \bar{X} , X' & \bar{X}' in the FS include pairs of $\bar{K}K$. Duality tells us:

$$\Gamma([b\bar{u}] \rightarrow u\bar{u}s\bar{u}) \simeq \Gamma(B^- \rightarrow \bar{K} + X_{S=0}) \quad (38)$$

$$\Gamma([b\bar{u}] \rightarrow u\bar{u}d\bar{u}) \simeq \Gamma(B^- \rightarrow X'_{S=0}) \quad (39)$$

Unitarity tell us that we can add pairs of $\bar{q}_i q_i$ to quark diagrams due to FSI. We *cannot* trust diagrams even semi-quantitatively to connect $u\bar{u}s\bar{q}_i q_i \bar{u}$ [$q_i = u, d, s$] with only $\bar{K}\pi\pi$ & $\bar{K}\bar{K}K$ or $u\bar{u}d\bar{q}_i q_i \bar{u}$ with $\pi\pi\pi$ & $\pi\bar{K}K$ including neutral pions & kaons to cancel asymmetry. What about hadronic FS $K^-\pi^+\pi^-\pi^0$, $K^-K^+K^-\pi^0$, $\bar{K}^0\pi^-\pi^+\pi^-$, $\bar{K}^0K^-K^+\pi^-$? Or 4π or $\pi\pi\bar{K}K$? Duality connects the worlds of (anti-)quarks with the one of hadrons, namely with two- & many-bodies FS; to be realistic we have to probe "regional" CP asymmetries with three- & four-body FS.

3.1.1 $B^\pm \rightarrow K^\pm\pi^+\pi^-$ vs. $B^\pm \rightarrow K^\pm K^+K^-$

The data of CKM suppressed B^+ decays show no surprising rates in PDG2015:

$$\begin{aligned}\text{BR}(B^+ \rightarrow K^+\pi^-\pi^+) &= (5.10 \pm 0.29) \cdot 10^{-5} \\ \text{BR}(B^+ \rightarrow K^+K^-K^+) &= (3.37 \pm 0.22) \cdot 10^{-5} .\end{aligned}\quad (40)$$

Operators O_+ & O_- give the same large weak phase from V_{ub} , while the penguin operator gives zero weak phase. O_{Peng} contributes to the branching ratios. Penguin diagrams give pictorials for FSI. However, they do not allow us to calculate their impact on exclusive transitions beyond have-waving arguments.

LHCb data show averaged CP asymmetries [37]:

$$\begin{aligned}\Delta A_{CP}(B^\pm \rightarrow K^\pm\pi^+\pi^-) &= +0.032 \pm 0.008_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.007_{\psi K^\pm} \\ \Delta A_{CP}(B^\pm \rightarrow K^\pm K^+K^-) &= -0.043 \pm 0.009_{\text{stat}} \pm 0.003_{\text{syst}} \pm 0.007_{\psi K^\pm}\end{aligned}\quad (41)$$

with 2.8σ & 3.7σ from zero. The sizes of these averaged asymmetries are not really surprising (unless one thinks more about interferences on these Dalitz plots); however it does not mean that we could really predict them. It is very interesting that they come with opposite sign due to CPT invariance.

LHCb data show regional CP asymmetries [37, 38]:

$$\begin{aligned}A_{CP}(B^\pm \rightarrow K^\pm\pi^+\pi^-)|_{\text{regional}} &= +0.678 \pm 0.078_{\text{stat}} \pm 0.032_{\text{syst}} \pm 0.007_{\psi K^\pm} \\ A_{CP}(B^\pm \rightarrow K^\pm K^+K^-)|_{\text{regional}} &= -0.226 \pm 0.020_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.007_{\psi K^\pm} .\end{aligned}\quad (42)$$

Regional CP asymmetries are defined by the LHCb collaboration: positive asymmetry at low $m_{\pi^+\pi^-}$ just below m_{ρ^0} ; negative asymmetry both at low and high $m_{K^+K^-}$ values. It should be noted the opposite signs in Eqs.(41,42). One needs more data about regional CP asymmetries – but also more thinking and better theoretical tools for strong FSI. One expects larger regional CP asymmetries here – but where and so large? Can it show the impact of broad resonances like $f_0(500)/\sigma$ and $K^*(800)/\kappa$? At least they give us highly non-trivial lessons about non-perturbative QCD. We have to wait from Belle II to probe $B^\pm \rightarrow K^\pm\pi^0\pi^0/K^\pm\bar{K}^0K^0$ (& $B^\pm \rightarrow K^\pm\eta^{(\prime)}\pi^0$).

3.1.2 $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ vs. $B^\pm \rightarrow \pi^\pm K^+ K^-$

Again the data of even more CKM suppressed B^+ decays show no surprising rates

$$\begin{aligned}\text{BR}(B^+ \rightarrow \pi^+ \pi^- \pi^+) &= (1.52 \pm 0.14) \cdot 10^{-5} \\ \text{BR}(B^+ \rightarrow \pi^+ K^- K^+) &= (0.50 \pm 0.07) \cdot 10^{-5}.\end{aligned}\quad (43)$$

One expects smaller rates of these FS based on the ‘experience’ from $\bar{B}_d \rightarrow K^- \pi^+$ vs. $\bar{B}_d \rightarrow \pi^+ \pi^-$ ones; indeed it is true given for the data. While the amplitudes of O_\pm operators are less suppressed above ones, $b \Rightarrow d$ penguin diagram is more suppressed than $b \Rightarrow s$ ones.

Data show *larger* CP asymmetries as discussed above in Eqs.(41,42) [39] (again with the opposite signs):

$$\begin{aligned}\Delta A_{CP}(B^\pm \rightarrow \pi^\pm \pi^+ \pi^-) &= +0.117 \pm 0.021_{\text{stat}} \pm 0.009_{\text{syst}} \pm 0.007_{\psi K^\pm} \\ \Delta A_{CP}(B^\pm \rightarrow \pi^\pm K^+ K^-) &= -0.141 \pm 0.040_{\text{stat}} \pm 0.018_{\text{syst}} \pm 0.007_{\psi K^\pm}.\end{aligned}\quad (44)$$

However comparing CP asymmetries show the surprising impact of penguin diagrams: in the SM one gets amplitudes $T(b \Rightarrow d)$; it also produces weak phase with V_{td} on the same *level* as V_{ub} in the SM. Again: re-scattering happens and affects sizably transitions – however where and how much?

Again CP asymmetries focus on small regions in the Dalitz plots [39, 38].

$$\begin{aligned}\Delta A_{CP}(B^\pm \rightarrow \pi^\pm \pi^+ \pi^-)|_{\text{regional}} &= +0.584 \pm 0.082_{\text{stat}} \pm 0.027_{\text{syst}} \pm 0.007_{\psi K^\pm} \\ \Delta A_{CP}(B^\pm \rightarrow \pi^\pm K^+ K^-)|_{\text{regional}} &= -0.648 \pm 0.070_{\text{stat}} \pm 0.013_{\text{syst}} \pm 0.007_{\psi K^\pm}.\end{aligned}\quad (45)$$

It should be noted also their signs in Eqs.(44,45). Again, does it show the impact of broad scalar resonances like $f_0(500)/\sigma$ and/or $K^*(800)/\kappa$?

3.1.3 Comparing with the literature for three-body FS

There is a large literature about three-body FS in suppressed decays of B^\pm and sizable ones for CP asymmetries including Dalitz plots. Obviously I agree we need to probe suppressed B decays with three-body FS with accuracy in general; however, I disagree with several statements giving in three articles [40, 41, 42]. Later I will discuss an important point, namely the connections with the B transitions with D & τ ones.

- In those articles no reference was given about earlier papers, like Refs.[23, 24, 13]; in two of them simulation was given for the possible impact of re-scattering. In [23] the connection of $B^\pm \rightarrow 3\pi, \pi \bar{K} K, K \pi \pi$ & $K \bar{K} K$. The importance of CPT invariance was emphasized in [24]; in [13] it was said that CPT invariance is ‘usable’. It was pointed out that U- & V-spin symmetries are connected by strong forces.
- One can compare $B^\pm \rightarrow K^\pm \pi^+ \pi^-$, $K^\pm \pi^0 \pi^0$, $K^\pm K^+ K^-$, $K^\pm \bar{K}^0 K^0$. In the future one wants to measure FS including $\eta^{(\prime)}$ and even think to include “constituent” gluons in their wave functions.

- The impact of FSI was discussed in general [15, 16, 17, 18].
- For probing weak decays of beauty mesons we have entered a new era, where we know that the SM gives basically the leading source of CP asymmetries. Therefore we need a refined parametrization of the CKM matrix for beauty (& charm) hadrons as pointed out above.
- I have said before that we should not forget the impact of CPT invariance in principle, but in charm transitions it is usable; we have learnt it is also usable for three- & four-body FS of beauty hadrons, as I said in [13]. For example, the data describe a surprising simple landscape, namely we find that $A_{CP}(B^+ \rightarrow K^+\pi^+\pi^-) \sim -A_{CP}(B^+ \rightarrow K^+K^+K^-)$ and $A_{CP}(B^+ \rightarrow \pi^+\pi^+\pi^-) \sim -A_{CP}(B^+ \rightarrow \pi^+K^+K^-)$ and the regional asymmetries are large.
- In my view it is not enough to fit the data in the best road; we have to use dispersion relations to understand the underlying dynamics as long as our analyses give us good results. The tools of dispersion relations have a successful & long history [22, 27] as mentioned in Sect.2.3. It shows the connection of Hadrodynamics & HEP; so far it has been applied to favored transitions of charm mesons.

3.2 Triple-product asymmetries

BaBar [43], Belle [44] and LHCb [45] have measured polarization amplitudes of $B^0 \rightarrow \phi K^*(892)$; the first two have measured also with other resonances. Their data have shown impact of re-scatterings, but have found no sign for CPV there. For example, LHCb analyses give $\Delta_{CP} = +0.015 \pm 0.032 \pm 0.005$. There is a puzzle:

- (a) Large direct CPV has been established ~ 10 & 20 % in $B^0 \rightarrow K^+\pi^-$ & $B_s^0 \rightarrow \pi^+K^-$, see Eq.(31).
- (b) Three-body FS in charged B decays produce CP asymmetries on the scale of $\sim 5 - 10$ % for averaged ones and ~ 50 % for regional ones.
- (c) Data show sizable impact of FSI and/or ND in two-body ones and large ones for three-body FS.
- (d) It makes sense that broad (scalar) resonances – like $f_0(500)/\sigma$ & $K^*(800)/\kappa$ – to give large impact on CP asymmetries on three-body FS – and even more for four-body FS, when one goes beyond moments. It means to probe semi-regional CP asymmetries as discussed above in Sect. 2.4 in general. It depends also on "judgment", which tools one can use.
- (e) I see no reason, why transitions for four-body FS can produce only small CP asymmetries. As again, the SM gives large weak phase with V_{ub} in general and V_{td} in special situation; on the other hand the landscapes for strong phases is very complex due to FSI as described in Sect. 2.4 with one-dimensional observables. Since it depend crucially on strong forces, we cannot produce predictions. My point is: I "paint" the situation and focus on correlation with different two- & four-body FS of B^0 , B^+ and B_s^0 .

3.2.1 Comparing with the literature about four-body FS

CPV in $\Delta B \neq 0$ has been established basically in two-body FS in 2001 - 2007. Some theoretical papers about four-body FS have been produced before 2001, but many followed the road from Ref.[28], namely with special situations $B \rightarrow V_a V_b \rightarrow h_1 h_2 h_3 h_4$ [46, 47]. However, our goals have changed after 2001: we have learnt that the SM gives at least the leading source for CPV in $\Delta B \neq 0$. Obviously we have learnt new lessons about the impact of QCD forces for exclusive decays. The goal for the future is to find signs of ND as a non-leading source for CPV and maybe even its features. It means we cannot focus only on special situations as said above; we have to probe four-body FS in general, or at least include broad resonances. We cannot quantitatively predict CP asymmetries in two- & four-body FS ways, but we cannot ignore four-body FS. Again, we cannot trust the lessons we got from diagrams. In particular re-scattering connects U- & V-spin symmetries. The measured averaged triple-product asymmetry in $B^0 \rightarrow \phi K^*(892)$ with $\Delta_{CP} = +0.015 \pm 0.032 \pm 0.005$ is consistent with zero CP asymmetry or also non-zero value for direct CPV; furthermore with more data we have to probe semi-regional CP asymmetries there and beyond, namely about $B^0 \rightarrow K^+ K^- K^+ \pi^-$. In general I want to emphasize to probe semi-regional asymmetries in other four-body FS with $B_{u,d,s}$.

Finally, I want to emphasize that the landscapes of $\Delta S \neq 0$ and $\Delta B \neq 0$ are quite different. Kaon amplitudes can be described with diagrams based on local operators, where chiral symmetry is a strong tools. However, the situations are quite different for B transitions for several reasons. The lessons we have learn from weak kaon decays cannot just transferred.

3.3 Short resume: Surprises & maybe some puzzles

These measured regional asymmetries are much larger than averaged ones in only charged three-body FS. Those show the impact of due to FSI and also much larger than two-body CPV. There are informations in several levels; some are expected, while others are more subtle or challenge our understanding of dynamics.

It is very interesting – but not surprising due to CPT invariance – that one finds CP asymmetries with opposite signs in FS with a pair of $\pi^+ \pi^-$ vs. $K^+ K^-$ in these three-body FS of B^\pm decays. Therefore one predicts averaged CP asymmetries with opposite signs $K^0 \bar{K}^0$ vs. $\pi^0 \pi^0$ (ignoring FS with η & η' to make it short). FSI connect U- and V-spin symmetries in weak decays.

Well known penguin diagrams $b \Rightarrow s$ compete well (or more) with tree diagram $b \rightarrow u \bar{u} s$ as shown about the rates $B^0 \rightarrow K^+ \pi^-$ vs. $B^0 \rightarrow \pi^+ \pi^-$ and CP asymmetries. There are several main points.

- The SM gives at least the leading source of measured CPV. Therefore we have to probe CPV for signs of ND as non-leading one.
- The impact of SM penguin diagrams $b \Rightarrow d$ should be more suppressed than $b \Rightarrow s$; the data about three-body FS semi-quantitatively show these rates. However

averaged CPV already seem to be sizably larger as you can see in Eqs.(41, 44). It describes a quite different situation in three-body FS than in two-body FS.

- Can the SM produce these very large regional ones or not? Do they suggest that our control over non-perturbative QCD is much less than we thought in many-body FS? Are there novel lessons about strong forces?
- The best fitted analyses often do not give the best information about underlying dynamics. The statements about Eqs.(42, 45) depend on the definition about "regional" asymmetries. Furthermore we have to include the impact of well-known resonances like $f_0(500)/\sigma$ & $K^*(800)/\kappa$ as long they produce analyses in accepted ways. It is subtle, and we have to think about correlations with other transitions.
- Comparing three- & four-body FS in the weak decays in beauty hadrons at least reveals novel lessons about non-perturbative QCD.
- Very short comments: measuring CP asymmetries in Λ_b^0 decays gives true challenges for the LHCb collab., and even more for Ξ_b ones. Belle II unlikely enter this competition.
- Evidence for CPV in $B^+ \rightarrow \bar{p}pK^+$ was found [48]. If it stands, it would be more than interesting or unusual: large regional CP asymmetry was found like also antibaryon-baryon FS. What about $B^+ \rightarrow pp\pi^+$ and $B^0 \rightarrow \bar{p}pK^+\pi^-$? Is it a novel lesson about strong dynamics about FS or a sign of ND as mentioned before [49].

Furthermore it is crucial to include *neutral* hadrons in the FS to understand the informations that data give us. I have pointed out before like at a recent paper [13]. It was discussed in details in Ref.[50] based on factorization model. I do not trust those models semi-quantitatively; here we pointed out the important impact of re-scattering. We know that re-scattering happens all the times and affect sizably asymmetries.

It seems that the duality between quarks and hadrons worlds are not close to local connections. Other symmetries (and their violations/limits) can limit the classes of hadrons involved. It might tell us that we are missing important information about underlying dynamics and have to think more. There are good chances that these 'prices' will be awarded with real 'prizes'.

4 $\Delta D \neq 0$ dynamics

Present data show us that neither indirect nor direct CPV have been found yet in charm hadrons. Finding CP asymmetries in charm hadrons – including charm *baryons* – in the future would be a pioneering achievement. The landscape of data is slim in particular about DCS ones, when one talks about CP asymmetries. It will change later from Belle II about $\Delta C \neq 0$ with D^0 & $D_{(s)}^+$ and Λ_c^+ (& $\Xi_c^{0,+}$) decays.

Non-leptonic FS are given mostly by two-, three- & four-body FS. The impact of CPT invariance is more obvious, but not trivial. The SM produces quite small CPV in SCS

transitions and basically zero on DCS ones, see Sect.2.1. We expect non-zero CPV in SCS transitions. We can calculate $c \Rightarrow u$ from intermediate b quark base on local operators, and they are tiny; yet intermediate s & d do *not* produced from short distance dynamics. Still we ‘expect’ they produce non-zero SCS amplitudes.

FSI affect DCS amplitudes sizably while penguins cannot do it. Furthermore the SM gives basically zero weak phase there. Thus DCS amplitudes give us wonderful places to probe the existence of ND and even its features – if we have enough data.

There is a well-known example about of the impact of re-scattering and its connection with U-spin symmetry. The data show the SCS ratio $\frac{\Gamma(D^0 \rightarrow K^+ K^-)}{\Gamma(D^0 \rightarrow \pi^+ \pi^-)} \sim 3$ vs. ”originally expected” ~ 1.4 based on (broken) U-spin symmetry. It was suggested that penguin diagrams might solve this puzzle about two-body FS and shows a ‘road’ for large violation of U-spin symmetry [51]. On the other hand one can compare the measured rates of two- and four-body FS with only charged hadrons: $\text{BR}(D^0 \rightarrow \pi^+ \pi^-) \simeq 1.4 \times 10^{-3}$ vs. $\text{BR}(D^0 \rightarrow K^+ K^-) \simeq 4 \times 10^{-3}$ and $\text{BR}(D^0 \rightarrow 2\pi^+ 2\pi^-) \simeq 7.4 \times 10^{-3}$ vs. $\text{BR}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \simeq 2.4 \times 10^{-3}$. Obviously the situation has changed very much from two- to four-body hadronic FS: the ratio of two-body FS $\sim 1.4/4 \sim 0.35$ changes to the ratio of four-body FS $\sim 7.4/2.4 \sim 3.1$ – i.e., by a factor ~ 10 . It is important – and not surprising – that FSI has changed the landscapes sizably. However their sum shows: $\frac{\text{BR}(D^0 \rightarrow K^+ K^-) + \text{BR}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-)}{\text{BR}(D^0 \rightarrow \pi^+ \pi^-) + \text{BR}(D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-)} \sim 0.73$; i.e., it is getting close to unity as one expects for inclusive ones in the world of (current) quarks due to m_d, m_s being very small on the scale of ~ 1 GeV. When one looks at diagrams, it shows the impact of FSI; however, getting numbers is another thing. The situation about CP asymmetries is much more complex with weak & strong phases than about rates; the connection of measured (or measurable) CP asymmetries with hadrons with amplitudes in the world of quarks is subtle. However, we cannot ignore four-body FS.

It has been suggested to probe U-spin symmetry by comparing amplitudes of $D^0 \rightarrow K^- \pi^+, K^+ K^-, \pi^+ \pi^-, K^+ \pi^-$ [52]. I quite *disagree*: effective transition amplitude with re-scattering connect not only charged FS mesons, but neutral ones. Furthermore many-body FS have large impact, see Sect.2.2.1; i.e., the differences between U- & V-spin symmetries in the world of hadrons are fuzzy, since they connect due to strong forces. If the U-spin violations are so small and therefore of the expansion of U-spin violations makes sense, I would see that is ‘luck’ so far in a special case – or we miss some important features of non-perturbative QCD.

Golden & Grinstein [7] were the first to focus on three-body FS in D decays using non-trivial theoretical tools. Now we have more refined theoretical tools like dispersion relations. Three-body FS with hadrons happen everywhere in the Dalitz plots; interferences appear in different locations.

Hadronic uncertainties in $c \Rightarrow u$ decays are discussed in Ref.[53], in particular about $D^0 \rightarrow \pi^+ \pi^-, \rho^+ \rho^-, \rho \pi$. I am somewhat disagree with some of their statements or at least their choice of words, namely ‘tree’ (T), ‘ W -exchanges’ (E) and three ‘penguin’ amplitudes ($P_{d,s,b}$). There are subtle, but important differences between diagrams, local & non-local operators; there are non-trivial challenges we have to face as I had said before.

Again the left sides of Eqs. (9,10) describe amplitudes of hadrons; the right sides deal with bound states of $\bar{q}_i q_j$.

First I give a comment or two about CP asymmetries in two-body FS that the LHCb collab. presented [54]:

$$A_{CP}(D^+ \rightarrow K_S K^+) = (0.03 \pm 0.17_{\text{stat}} \pm 0.14_{\text{syst}})\% \quad (46)$$

$$A_{CP}(D_s^+ \rightarrow K_S \pi^+) = (0.38 \pm 0.46_{\text{stat}} \pm 0.17_{\text{syst}})\% \quad (47)$$

The data are consistent with zero values, but also $\mathcal{O}(10^{-3})$. One can combine also

$$A_{CP}(D^+ \rightarrow K_S K^+) + A_{CP}(D_s^+ \rightarrow K_S \pi^+) = (0.41 \pm 0.49_{\text{stat}} \pm 0.26_{\text{syst}})\% \quad (48)$$

Does it show the impact of U-spin symmetry in weak decays? Or does it show the scale of SCS decays at $\mathcal{O}(10^{-3})$? What about a combination of both? The LHCb collab. has probed $D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$ with time-integrated CPV [54]:

$$\begin{aligned} A_{CP}(D^0 \rightarrow K^+ K^-) &= (-0.06 \pm 0.15_{\text{stat}} \pm 0.10_{\text{syst}})\% \\ A_{CP}(D^0 \rightarrow \pi^+ \pi^-) &= (-0.20 \pm 0.19_{\text{stat}} \pm 0.10_{\text{syst}})\% \\ \Delta A_{CP} &= (+0.14 \pm 0.16_{\text{stat}} \pm 0.08_{\text{syst}})\% \end{aligned} \quad (49)$$

What about the same basic questions as said above: we have to deal with complex situations. I emphasize that we cannot focus only on two-body FS. In my view the main point is to truly probe CP asymmetries in three- & four-body FS, and we cannot ignore FSI between U- & V-spin symmetries; i.e., we cannot simply applying U-spin symmetry.

Actually we know that three- & four-body FS describe larger parts of the weak decays than two-body ones. Those show a more ‘complex’ landscape to combine weak & strong phases that produce CP asymmetries. Furthermore those data of the rates do *not* follow the same pattern as we can see from PDG2015: D^0 vs. D^+ vs. D_s^+ for SCS & DCS and three- vs. four-body FS. It shows the sizable impact of FSI. We should not be surprised, but we cannot predict theirs even in semi-quantitative ways. Finally CPT invariance is ‘usable’ in D^0 vs. \bar{D}^0 , D^+ vs. D^- and D_s^+ vs. D_s^- transitions. As said above, it is surprising that it also ‘usable’ for B^+ vs. B^- decays. Of course, CPT invariance is realized both with charged and neutral pions & kaons (plus $\eta^{(\prime)}$).

Again, I use the word of ‘painting’ the FS as the main point: the landscapes for SCS & DCS decays with three- & four-body FS are complex and different patterns about D^0 , D^+ , D_s^+ & Λ_c^+ . It shows the impact of FSI; therefore I do not discuss the details of the rates.

4.1 SCS decays of D^0 , D^+ & D_s^+ with three- & four-body FS

PDG 2015 gives rates for three-body FS that can be measured by LHCb in the future, namely mostly with charged hadrons in the FS:

$$\text{BR}(D^0 \rightarrow K^+ K_S \pi^-) \sim 2.2 \cdot 10^{-3} \quad , \quad \text{BR}(D^0 \rightarrow K^- K_S \pi^+) \sim 3.6 \cdot 10^{-3}$$

$$\text{BR}(D^0 \rightarrow K^+ K^- \pi^0) \sim 3.4 \cdot 10^{-3} \quad , \quad \text{BR}(D^0 \rightarrow \pi^+ \pi^- \pi^0) \sim 14.7 \cdot 10^{-3} \quad (50)$$

$$\text{BR}(D^+ \rightarrow \pi^+ \pi^+ \pi^-) \sim 3.3 \cdot 10^{-3} \quad , \quad \text{BR}(D^+ \rightarrow \pi^+ K^+ K^-) \sim 10 \cdot 10^{-3} \quad (51)$$

$$\text{BR}(D_s^+ \rightarrow K^+ \pi^+ \pi^-) \sim 6.6 \cdot 10^{-3} \quad , \quad \text{BR}(D_s^+ \rightarrow K^+ K^+ K^-) \sim 0.22 \cdot 10^{-3} \quad (52)$$

I have no predictions for these rates or quantitative one for CP asymmetries beyond saying that averaged CP asymmetry are of the level of $\mathcal{O}(0.1)\%$ from the SM. In the future we have to probe Dalitz plots with the impact of FSI on regional CP asymmetries and their correlations due to CPT invariance⁷. It was discussed in Ref.[25] with simulations of $D^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ and $D^\pm \rightarrow \pi^\pm K^+ K^-$ with small weak phases and sizable resonances phases in the world of hadrons.

Here I want to emphasize that we see quite different landscapes for charm mesons mostly with charged FS. For D^0 decays one sees that the combination of a pair of $\bar{K}K$ vs. without them are somewhat similar, while these pair larger than without them for D^+ ; on the other hand D_s^+ decays will us much smaller rates with a pair of $\bar{K}K$ than without. Those show the impact of FSI in different ways. There are good reasons why to compare binned "fractional asymmetries" vs. "significance" [25] vs. "un-binned" ones [26]. A short comment: it might show the different impact of WA on rates and effective phases for D^+ vs D_s^+ .

Again, it should not be the final step. Subtle, but powerful tools have not been applied yet like dispersion relations [22, 27, 55]; those are based on low energy collisions between two hadrons in the future. We have the tools to do that – but it takes time. It is always wonderful to get more ideas, but there is no reason to wait for novel ideas.

I add a comment that connect with FSI: LHCb has measured SCS $D^0 \rightarrow \pi^+ \pi^- \pi^0$ [56]. Their paper starts with "The decay $D^0 \rightarrow \pi^- \pi^+ \pi^0$... proceeds via a singly Cabibbo suppressed $c \rightarrow d u \bar{d}$ transition with a possible admixture from a penguin amplitude." It seems to ignore FSI from $c \rightarrow s u \bar{s}$ diagram; I see no good reason for that. Above in Sect.2.2.1 I have discussed the impact of re-scattering in general; here we cannot ignore refined local operators $c \rightarrow s u \bar{s}$. Of course, one measure the rates & asymmetries of $D^0 \rightarrow 3\pi$ & $D^0 \rightarrow \pi \bar{K} K$ in the world of hadrons. My point is to connect amplitudes in the world of quarks. Duality is not trivial at all.

Data above show that $\Gamma(D^0 \rightarrow \pi^+ \pi^- \pi^0)$ is larger than single $\Gamma(D^0 \rightarrow \pi \bar{K} K)$; however we get $\sum \Gamma(D^0 \rightarrow \pi \bar{K} K) \sim \Gamma(D^0 \rightarrow 3\pi)$.

Focusing on four-body FS with at most one neutral mesons I list FS that LHCb collaboration can measure.

$$\text{BR}(D^0 \rightarrow 2\pi^+ 2\pi^-) \sim 7.5 \cdot 10^{-3} \quad , \quad \text{BR}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \sim 2.4 \cdot 10^{-3} \quad (53)$$

$$\text{BR}(D^+ \rightarrow K^+ K_S \pi^+ \pi^-) \sim 1.7 \cdot 10^{-3} \quad , \quad \text{BR}(D^+ \rightarrow K^- K_S \pi^+ \pi^+) \sim 2.3 \cdot 10^{-3}$$

$$\text{BR}(D^+ \rightarrow \pi^+ \pi^- \pi^+ \pi^0) \sim 11.7 \cdot 10^{-3} \quad (54)$$

$$\text{BR}(D_s^+ \rightarrow K_S \pi^+ \pi^- \pi^+) = (3.0 \pm 1.1) \cdot 10^{-3} \quad (55)$$

The pattern is quite different from the three-body FS: four pions produce the leading source so far.

⁷Of course these correlations can be satisfied in other FS with neutral mesons.

T-odd momenta has been measured in $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ [57]:

$$A_T = (-7.18 \pm 0.41 \pm 0.13) \cdot 10^{-3} \quad , \quad \bar{A}_T = (-7.55 \pm 0.41 \pm 0.12) \cdot 10^{-3}$$

$$a_{CPV}^{T\text{-odd}} = (0.18 \pm 0.29 \pm 0.04)\% \quad (56)$$

It shows sizable impact of FSI: $A_T \neq 0 \neq \bar{A}_T$. On the other hand, no CP violation has been found yet. From these data I learn, namely that is just the beginning:

- One has to probe semi-regional CPV in $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ with different tools as discussed above in Sect. 2.4 in general.
- One can follow the somewhat roads for $D^0 \rightarrow 2\pi^+ 2\pi^-$. There is a complication, namely how to differentiate between $2\pi^\pm$.
- With more data one can think about comparing the informations from $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ vs. $D^0 \rightarrow 2\pi^+ 2\pi^-$ based on CPT invariance.
- We have to follow the same ‘roads’ in smart ways in $D^+ \rightarrow K_S \pi^+ K^\pm \pi^\mp$ and $D_s^+ \rightarrow K^+ \pi^+ \pi^- \pi^0 / K^+ K^+ K^- \pi^0 / K_S \pi^+ \pi^- \pi^+ / K_S K^+ K^- \pi^+$.

Again, I want to emphasize that drawing penguin diagrams is one thing, but trust the numbers they give is quite different.

4.2 DCS decays of D^0 , D^+ & D_s^+ with three- & four-body FS

For theorists the landscapes are much simpler for the impact of ND, since the SM background is close to zero with $c \rightarrow ud\bar{s}$ with $\Delta C = 1$ & $\Delta S = -1$. Furthermore there are only two refined tree operators, and penguin diagrams cannot contribute; however re-scattering does contribute sizably. Of course we need huge numbers of charm hadrons.

Present data from PDG15 are even slimmer:

$$\text{BR}(D^0 \rightarrow K^+ \pi^-) \sim 1.5 \cdot 10^{-4} \quad (57)$$

Two-body FS sets the scale.

$$\text{BR}(D^0 \rightarrow K^+ \pi^- \pi^0) \sim 3.1 \cdot 10^{-4} \quad (58)$$

$$\text{BR}(D^+ \rightarrow K^+ \pi^+ \pi^-) \sim 5.5 \cdot 10^{-4} \quad , \quad \text{BR}(D^+ \rightarrow 2K^+ K^-) \sim 0.90 \cdot 10^{-4} \quad (59)$$

$$\text{BR}(D_s^+ \rightarrow 2K^+ \pi^-) \sim 1.3 \cdot 10^{-4} \quad (60)$$

So far there are no real surprise in the landscape; of course there are sizable experimental uncertainties. Impact of ND can hide in $D^0 \rightarrow K^0 \pi \pi$ due to Cabibbo favored $D^0 \rightarrow \bar{K}^0 \pi \pi$ transitions.

On the other hand CP asymmetries can be probed in these $D^+ \rightarrow K^+ \pi^+ \pi^- / K^+ K^+ K^-$ and $D_s^+ \rightarrow K^+ K^+ \pi^-$, where the SM gives hardly any background; of course, we need more data. In the future we can measure ”fractional asymmetry”, ”significance” or ”unbinned” Dalitz plot [23, 25, 26].

So far we have virginal landscape for four-body FS, except:

$$\text{BR}(D^0 \rightarrow K^+\pi^+2\pi^-) \sim 2.6 \cdot 10^{-4} \quad (61)$$

LHCb should be able to measure averaged and practical definitions of regional CP asymmetries as discussed in Sect.2.4 in $D^+ \rightarrow K^+\pi^-\pi^+\pi^0, K^+K^-K^+\pi^0, K_S\pi^+\pi^+\pi^-$ and $D_s^+ \rightarrow K^+K^+\pi^-\pi^0, K^+K_S\pi^+\pi^-, K^+K^+\pi^-\eta$ for second steps.

4.3 General comments about CP asymmetries in charm baryons

Charm baryons might turn out to be the ‘Poor Princesses’ [58] for establishing CPV in baryons’ decays and showing the impact of ND there. It is very important to probe correlations between Λ_c^+, Ξ_c^+ & Ξ_c^0 decays in different levels ⁸. For DCS one gets WA diagram $[cs] \Rightarrow du$, while $[cd] \Rightarrow du$ and $[cs] \Rightarrow su$ for SCS. Their impacts depend on their wave functions.

It is not surprising that the Cabibbo favored $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay produces sizable branching ratio:

$$\text{BR}(\Lambda_c^+ \rightarrow pK^-\pi^+) = (6.84_{-0.40}^{+0.32}) \cdot 10^{-2} \quad (62)$$

LHCb will be able to measure this with accuracy soon; it can be used to calibrate branching ratios. Finding CP asymmetry there, it would be a miracle. To be realistic: one probes production asymmetry in pp collisions by measuring the ratio of $\bar{\Lambda}_c^- \rightarrow \bar{p}K^+\pi^-$ vs. $\Lambda_c^+ \rightarrow pK^-\pi^+$ ⁹. Thus one can calibrate the branching ratios of suppressed decays, when one probes CP asymmetries as discussed below.

The landscape is slim, when one wants to describe it in a ‘positive’ way:

$$\text{BR}(\Lambda_c^+ \rightarrow p\pi^+\pi^-) = (4.7 \pm 2.5) \cdot 10^{-3} \quad (63)$$

$$\text{BR}(\Lambda_c^+ \rightarrow pK^+K^-) = (1.1 \pm 0.4) \cdot 10^{-3} \quad (64)$$

It is not clear what we can learn from these numbers about underlying dynamics. Small CP asymmetries are most likely to appear *both* in $\Lambda_c^+ \rightarrow p\pi^+\pi^-$ with $p\rho^0, p\sigma$ etc. and $\Lambda_c^+ \rightarrow pK^+K^-$ with $p\phi, \Lambda^*K$ etc. As before, regional CP asymmetries are usually much larger than averaged asymmetries in three-body final states.

As the second step we have to probe Dalitz plots in ‘model insensitive’ ways, but again not as a final step. One has to be realistic: to extract the dynamics information underlying the finite data one has to apply quantitative theoretical analyses based on ”judgment”.

The present limits for these DCS decays are in the region one expects:

$$\text{BR}(\Lambda_c^+ \rightarrow pK^+\pi^-) < 3.1 \cdot 10^{-4} . \quad (65)$$

⁸When one understands the political landscape in Bavaria and the rest of Germany, one might see the analogy of $\Xi_c^+ = [csu]$ vs. $\Lambda_c^+ = [cdu]$.

⁹LHCb might be able also measure $\Lambda_c^+ \rightarrow p\bar{K}^0 \Rightarrow pK_S$ with accuracy. We know that indirect CP violation in $K^0 - \bar{K}^0$ oscillations has been measured by $\text{Re } \epsilon_K$; it checks experimental (un)certainly.

One has to remember that the landscapes of $\Lambda_c^+ \rightarrow pK^-\pi^+$ vs. $\Lambda_c^+ \rightarrow pK^+\pi^-$ are quite different due to contributions from both narrow & broad resonances. It is less complex than for SCS ones, since they are described by two refined local operators without penguin diagrams. Beyond the need for large numbers of Λ_c^+ the main challenges is to deal with production asymmetries in pp collisions.

As mentioned above, SM produces only one quark amplitude for DCS transitions; therefore SM cannot produce CP asymmetry. Furthermore the size of SM amplitudes are very much suppressed; thus it gives more sensitivity to the impact of ND.

In particular, one can analyze $\Lambda_c^+ \rightarrow pK^+\pi^-$ vs. $\bar{\Lambda}_c^- \rightarrow \bar{p}K^-\pi^+$ for CP violation and compare with CF $\Lambda_c^+ \rightarrow pK^-\pi^+$ & $\bar{\Lambda}_c^- \rightarrow \bar{p}K^+\pi^-$ to learn about the impact of FSI. Of course, one has to differentiate $K^+\pi^-$ from $K^-\pi^+$ in the Λ_c^+ decays despite the huge difference in their branching ratios. These DCS decays have not been found yet. On the other hand one can hope for significant CP violation in the DCS transitions.

The Λ_c final states include pK^* , $p\kappa$, N^*K , $\Delta^{(*)}K$, $\Lambda^*\pi$ etc. – i.e., numerous states that give us lessons about the existence of ND and its features due to several reasons. These are qualitative and at most a semi-quantitative comments. Quantitative theoretical works will happen based on dispersion relations, but they will take more efforts and time (in particular for pK & $p\pi$ states).

Of course crucial jobs will be done by Belle II, and more refined theoretical technologies have to be applied to understand the data about the underlying dynamics.

5 Summary

I talk mostly about strategies in heavy flavor dynamics and discuss the landscapes.

- We know that the SM gives us at least the leading source of CPV in the transitions of beauty mesons (and $K_L \rightarrow \pi\pi$). Therefore we have to probe those transitions about non-leading sources; at least we learn about non-perturbative QCD.
- It is very important to establish CPV in charm hadrons. Furthermore we expect $\mathcal{O}(0.001)$ averaged CP asymmetries in SCS decays and basically zero in DCS ones from the SM. It is due to new parametrization of the CKM matrix [12], which is less obvious, but more consistent. DCS amplitudes might be an excellent hunting for ND – if we have enough data and apply refined analyses.
- To find CP asymmetries in beauty & charm *baryons* is an important achievement. At least it gives novel lessons about strong dynamics.
- On the theoretical side no large progress has been established in understanding fundamental dynamics in many-body FS. On the other hand data from the LHCb collaboration give theorists excellent reason to enter the ‘game’. There are two different ‘cultures’, namely Hadrodynamics and HEP. It is not trivial to combine their tools; however, it is crucial to apply in new landscapes with accuracy.

- I see *no reason* why two-, three- & four-body FS follow the same pattern in the decays both of beauty & charm hadrons. Actually data show different patterns for rates and CP asymmetries. Based on experience we know that FSI/re-scattering has impact, in particular about connections with U- & V-spin symmetries.
- It is important to measure *correlations* between charm and beauty hadrons (& τ leptons), *not* focus on one or two ‘golden’ tests.
- It is not a good idea to base our conclusions coming from the best fitting of the data available; we need deeper thinking ¹⁰.
- We have to probe regional CP asymmetries in many-body FS, namely three- & four-body FS. It is an excellent achievement by the LHCb by measuring regional CP asymmetries in $B^- \rightarrow K^- \pi^+ \pi^- / K^- K^+ K^- / \pi^- \pi^+ \pi^- / \pi^- K^+ K^-$ on the experimental side. We have to use tested tools based on dispersion relations etc., which shows collaborations of experimenters & theorists backward & forward ¹¹.
- The impacts of penguin diagrams are complex. In the SM one predicts inclusive *beauty* decays. For exclusive ones we need help from other tools like chiral symmetry & dispersion relations. There are other problems about penguin diagrams in charm transitions. They give pictorials for SCS ones including FSI, but not much more; they cannot produce DCS amplitudes.
- The measured (or measurable) rates and their CP asymmetries are shown on the left side of Eq.(11). On the right side it shows the amplitudes that are based on the theory of the SM and/or ND (assuming CPT invariance). The connections of the worlds of hadrons vs. quarks (& gluons) are complex; i.e., the connections of the diagrams with underlying dynamics are not straightforward – namely the impact of re-scattering $a \rightarrow b + c$, whether one uses amplitudes of hadrons or quarks.

Here I had focused on three- & four-body FS with only charged ones and maybe also with one neutral hadron, where LHCb has measured and will continue. It is very important to probe FS with more neutral ones as Belle II will do it – but it will not happen very soon.

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¹⁰It was suggested by Stephan Ellis from the University of Washington in a very different situation, namely to probe internal structures of jets with “pruning” [59]; however the main point is the same, namely “judgment”.

¹¹One can learn from the history of art: “It is better to imitate ancient than modern work.” Leonardo da Vinci(1452 - 1519).

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